**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

🡪 # Find Z-Scores at X=50; Z = (X - µ) / σ

Z=(50-45)/8

Z=0625

# Find probability P(X>50) = 1-stats.norm.cdf(abs(z\_score))

1-stats.norm.cdf(abs(0.625))

**0.266**

**N**

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

🡪 We have a normal distribution with = 38 and = 6. Let X be the number of employees. So according to question

a) *# More employees at the processing center are older than 44 than between 38 and 44.*

*# p(X>44); Employees older than 44 yrs of age*

1**-**stats**.**norm**.**cdf(44,loc**=**38,scale**=**6)

0.15865525393145707

*# p(38<X<44); Employees between 38 to 44 yrs of age*

stats**.**norm**.**cdf(44,38,6)**-**stats**.**norm**.**cdf(38,38,6)

0.3413447460685429

Therefore the statement that More employees at the processing center are older than 44 than between 38 and 44 is TRUE.

b) *# B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.*

*# P(X<30); Employees under 30 yrs of age*

stats**.**norm**.**cdf(30,38,6)

0.09121121972586788

*# No. of employees attending training program from 400 nos. is N\*P(X<30)*

400**\***stats**.**norm**.**cdf(30,38,6)

36.484487890347154

Therefore the statement B of the question is also TRUE.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

🡪 As we know that if X <" N(µ1, Ã 1^2 ), and Y <" N(µ2, Ã 2^2 ) are two independent random variables then X + Y <" N(µ1 + µ2, Ã 1^2 + Ã 2^2 ) , and X " Y <" N(µ1 " µ2, Ã 1^2 + Ã 2^2 ) .

Similarly if Z = aX + bY , where X and Y are as defined above, i.e Z is linear combination of X and Y , then Z <" N(aµ1 + bµ2, a^2Ã 1^2 + b^2Ã 2^2 ).

Therefore in the question

2X1~ N(2 u,4 Ã ^2) and

X1+X2 ~ N(µ + µ, Ã ^2 + Ã ^2 ) ~ N(2 u, 2Ã ^2 )

2X1-(X1+X2) = N( 4µ,6 Ã ^2)

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

­🡪 Since we need to find out the values of a and b, which are symmetric about the mean, such that the probability of random variable taking a value between them is 0.99, we have to work out in reverse order.

**from** scipy **import** stats

**from** scipy.stats **import** norm

stats**.**norm**.**interval(0.99,100,20)

(48.4834, 151.5165)

So, option D is correct.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

🡪import numpy as np

from scipy import stats

from scipy.stats import norm

# Mean profits from two different divisions of a company = Mean1 + Mean2

Mean = 5+7

print('Mean Profit is Rs', Mean\*45,'Million')

-Mean Profit is Rs 540 Million

# Variance of profits from two different divisions of a company = SD^2 = SD1^2 + SD2^2

SD = np.sqrt((9)+(16))

print('Standard Deviation is Rs', SD\*45, 'Million')

-Standard Deviation is Rs 225.0 Million

# A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

print('Range is Rs',(stats.norm.interval(0.95,540,225)),'in Millions')

**-Range is Rs (99.00810347848784, 980.9918965215122) in Millions**

# B. Specify the 5th percentile of profit (in Rupees) for the company

# To compute 5th Percentile, we use the formula X=μ + Zσ; wherein from z table, 5 percentile = -1.645

X= 540+(-1.645)\*(225)

print('5th percentile of profit (in Million Rupees) is',np.round(X,))

**-5th percentile of profit (in Million Rupees) is 170.0**

# C. Which of the two divisions has a larger probability of making a loss in a given year?

# Probability of Division 1 making a loss P(X<0)

stats.norm.cdf(0,5,3)

**-0.0477903522728147**

# Probability of Division 2 making a loss P(X<0)

stats.norm.cdf(0,7,4)

**0.040059156863817086**